UNIT - III

Laplace Transforms.

Let $f(t)$ be a given function. and defined all +ve values of 't'. Then the laplace transform of $f(t)$ is denoted by $L\{f(t)\}$ or $\bar{f}(s)$

$$L\{f(t)\} \text{ (or) } \bar{f}(s) = \int_0^\infty e^{-st} f(t) \cdot dt$$

- Elementary function:-

$f(t)$ under Laplace transform $L\{f(t)\}$

S.NO	$f(t)$	$L\{f(t)\}$ or $\bar{f}(s)$	
1.	1	$1/s$	
2.	K : (where K is const)	K/s	
3.	t^n	$\frac{n!}{s^{n+1}}$	
4.	t^n (n is or -ve)	$\frac{\Gamma(n+1)}{s^{n+1}}$	
5.	$\sin at$	$\frac{a}{s^2+a^2}$	
6.	$\cos at$	$\frac{s}{s^2+a^2}$.	

7. e^{at} $\quad \frac{1}{s-a}$

8. e^{-at} $\quad \frac{1}{s+a}$

9. $\sinh at$ $\quad \frac{a}{s^2-a^2}$

10. $\cosh at$ $\quad \frac{s}{s^2-a^2}$

11. t $\quad \frac{1}{s^2}$

12. $\sqrt{t}$ $\quad \frac{\sqrt{\pi}}{2s^{3/2}}$

13. $1/\sqrt{t}$ $\quad \sqrt{\pi/s}$.

① $L\{e^{2t}+4t^3-2\sin 3t+3\cos 3t\}$

$L\{e^{2t}\}+L\{4t^3\}-L\{\sin 3t\}+L\{3\cos 3t\}$

$\frac{1}{s-2}+4L\{t^3\}-2L\{\sin 3t\}+3L\{\cos 3t\}$

$\frac{1}{s-2}+\frac{4(3!)}{s^4}-2\frac{(3)}{s^2+9}+3\left(\frac{s}{s^2+9}\right)$

$\frac{1}{s-2}+\frac{24}{s^4}-\frac{6}{s^2+9}+\frac{3s}{s^2+9}$.

$L\{e^{at}\}=\frac{1}{s-a}$

$$L\{t^n\} = \frac{n!}{s^{n+1}}$$

$$L\{\sin at\} = \frac{a}{s^2+a^2}$$

$$L\{\cos at\} = \frac{s}{s^2+a^2}$$

$$L\{k\} = k/s$$

② Find the L transform of $\cos^3 2t$.

$L\{\cos^3 2t\}$. $\cos^3 x = \frac{\cos 3x + 3\cos x}{4}$ $x = 2t$

Sol:-

$$L\left\{\frac{\cos 3(2t) + 3\cos(2t)}{4}\right\}$$

$$\frac{1}{4}\left\{L(\cos 6t) + 3L(\cos 2t)\right\}$$

$$\frac{1}{4}\left[\frac{s}{s^2+36} + 3\left(\frac{s}{s^2+4}\right)\right]$$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

$$\cos^3 A = \frac{\cos 3A + 3\cos A}{4}$$

~~formula:~~

Problems:-

① find the $L\left(e^{-t}\cos 2t\right)$.

Given,

Now $f(t) = \cos 2t$

$L(f(t)) = L\{\cos 2t\}$

$$\boxed{\bar{f}(s) = \frac{s}{s^2+4}}$$

Apply first shifting theorem:

$$L\left(e^{at} f(t)\right) = f(s-a) = \left[\bar{f}(s)\right]_{s\to s-a}$$

$$L\left(e^{-t}\cos 2t\right) = f(s+1) = \left[\frac{s}{s^2+4}\right]_{s\to s+1}$$

$$= \frac{s+1}{(s+1)^2+4} \to \frac{s+1}{s^2+2s+5}$$

$$L\left(e^{-t}\cos 2t\right) = \left(\frac{s}{s^2+4}\right)_{s\to s+1}.$$

② $e^{-3t}\left(2\cos 5t - 3\sin 5t\right)$.

$f(t) = 2\cos 5t - 3\sin 5t$

$L(f(t)) = L\left(2\cos 5t - 3\sin 5t\right)$

$= L\left(2\cos 5t\right) - L\left(3\sin 5t\right)$

$= 2L\left(\cos 5t\right) - 3L\left(\sin 5t\right)$

$$\bar{f}(s) = 2\left[\frac{s}{s^2+25}\right] - 3\left[\frac{5}{s^2+25}\right]$$

Apply first shifting theorem.

$$L\{e^{at}f(t)\} = f(s-a) = [\bar{f}(s)]_{s\to s-a}$$

$$L\{e^{-3t}(2\cos 5t - 3\sin 5t)\} = f(s+3)$$

$$= \left[\frac{2s}{s^2+25} - \frac{15}{s^2+25}\right]_{s\to s+3}$$

$$= \frac{2(s+3)}{(s+3)^2+25} - \frac{15}{(s+3)^2+25}$$

(3) Find the Laplace transform of

$$L\left(\sqrt{t}\cdot e^{-3t}\right).$$

$$= L(\sqrt{t}) = \frac{\sqrt{1/2+1}}{s^{1/2+1}}$$

$$= \frac{\sqrt{3/2}}{s^{3/2}}$$

$$= \frac{(3/2-1)\sqrt{3/2-1}}{s^{3/2}}$$

$$L(t^n) = \frac{\sqrt{n+1}}{s^{n+1}}$$

$$= \frac{1/2\,\Gamma(1/2)}{s^{3/2}} \Rightarrow \frac{1/2\sqrt{\pi}}{s^{3/2}} = \frac{\sqrt{\pi}}{2s^{3/2}}$$

$$= L(\sqrt{t}) = \frac{\sqrt{\pi}}{2s^{3/2}} = \bar{f}(s).$$

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Second Shifting Theorem :-

If laplace of $f(t)$

$$\boxed{L\{f(t)\} = \bar{f}(s)}$$ and

$$\boxed{g(t) = \begin{cases} f(t-a), & t > 0 \\ 0, & t < a \end{cases}}$$

Then

$$\boxed{L\{g(t)\} = e^{-as} \bar{f}(s)}$$

(or) If $L\{f(t)\} = \bar{f}(s)$ and $a > 0$ Then $\boxed{L\{f(t-a) \cdot u(t-a)\} = e^{-as} \bar{f}(s)}$

Q Find the Laplace transform of $\cancel{(t-2)^3}$

$(t-2)^3 \cdot u(t-2)$

Let $f(t) = t^3$.

$L\{f(t)\} = L(t^3)$

$= \frac{3!}{s^4} = \frac{6}{s^4}$

Annlying 2nd Shifting t

$\therefore L\{g(t)\} = L\{(t-2)^3 u(t-2)\} = e^{-2s} \cdot \frac{6}{s^4}$

$L\{-2\}$

$\frac{-2}{s}$

27/30

Q Find $L\{3\cos 4(t-2)\, u(t-2)\}$

let $f(t) = 3\cos 4t$

$\bar{f}(t) = L\{3\cos 4t\} =$

$$\bar{f}(s) = 3 \cdot \frac{s}{s^2+16}$$

Applying 2nd shifting,

$$\therefore L\{3\cos 4(t-2)\, u(t-2)\} = \frac{e^{-2s} \cdot 3s}{s^2+16}.$$

Q Find $L\{g(t)\}$ $g(t) = \begin{cases} \cos(t - \frac{\pi}{3}) & t > \pi/3 \\ 0 & t < \pi/3 \end{cases}$

Let $f(t) = \cos t$

$$\bar{f}(s) = \frac{s}{s^2+1}$$

Applying 2nd shifting,

$$L\{g(t)\} = e^{-\frac{\pi}{3}s}\left(\frac{s}{s^2+1}\right)$$

Change of Scale Property.

If $L\{f(t)\} = \bar{f}(s)$ then $L\{f(at)\} = \frac{1}{a}\bar{f}(s/a)$.

Q $L\{f(t)\} = \frac{9s^2 - 12s + 15}{(s-1)^3} = \bar{f}(s)$ find $L\{f(3t)\}$

by using change of scale property.

$$L\{f(3t)\} = \frac{1}{3} \frac{9\left(\frac{s}{3}\right)^2 - 12\left(\frac{s}{3}\right) + 15}{\left(\frac{s}{3} - 1\right)^3} = \frac{9(s^2 - 4s + 15)}{(s-3)^3}$$

$$= 3\left(\frac{s^2 - 4s + 15}{(s-3)^3}\right)$$

Q Find $L\{e^{-3t}\sinh 3t\}$. use change of scale property.

$$L\{f(t)\} = \sinh t \quad \text{——} ①$$

By change of scale property ← $(a = 3)$

$$\cancel{L\{e^{-3t}\sinh 3t\} = \frac{1}{(s-3)^2-1}} \qquad \frac{1}{3}\ \frac{1}{(\frac{s}{3}-9)^2-1}$$

$$L\{f(3t)\} = \frac{1}{3} f'(s/3)$$

$$L\{\sinh 3t\} = \frac{1}{3}\ \frac{1}{(\frac{s}{3})^2-1}$$

$$= \frac{3}{s^2-3^2}$$

By 1st shifting ← $(a = -3)$

$$L\{e^{-3t}\sinh 3t\} = \frac{3}{(s+3)^2-9}$$

(or)

① ⇒ By 1st shifting ← $(a = -1)$

$$L\{e^{-t}\sinh t\} = \frac{1}{(s+3)^2-1}$$

By change of scale property $(a = 3)$

$$L\{f(3t)\} = \frac{1}{3} f'(s/3)$$

$$L\{e^{-3t}\sinh 3t\} = \frac{1}{3}\ \frac{1}{(\frac{s}{3}+3)^2-1} = \frac{3}{(s+3)^2-9}$$

Q If $L\{f(t)\} = \frac{1}{s} e^{-1/s} = \bar{f}(s)$

Prove that

$$L\{e^{-t} f(3t)\} = \frac{e^{-3/s+1}}{s+1}$$

By change of scale property.

$$L\{f(3t)\} = \frac{1}{3} \bar{f}(s/3)$$

$$= \frac{1}{\cancel{3}} \frac{1}{(s/\cancel{3})} e^{-1/(s/3)}$$

$$L\{f(3t)\} = \frac{e^{-3/s}}{s} \quad (s-a)$$

By first shifting. $a = -1$

$$L\{e^{-t} f(3t)\} = \frac{e^{-3/(s+1)}}{s+1}$$

Hence Proved.

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Laplace Transform of Integral.

If $L\{f(t)\} = \bar{f}(s)$ then

$$L\left\{\int_0^t f(t)\,dt\right\} = \frac{1}{s} \times \bar{f}(s)$$

Q Find $L\left\{ \int_0^t \cos h t \, dt \right\}$

$f(t) = \cosh t$

$\bar{f}(s) = \dfrac{s}{s^2-1}$ Laplace integral.

By Laplace integral

$L\left\{ \int_0^t f(t)\,dt \right\} = \dfrac{1}{s} \dfrac{s}{s^2-1}$

$= \dfrac{1}{s^2-1}$

Find $L\left\{ \int_0^t e^{-t} \cos t \, dt \right\}$.

$f(t) = e^{-t} \cos t$

1^{st} shifing

$L\left\{ e^{-t} \cos t \right\} = \left[\dfrac{s}{s^2+1} \right]_{s \to s+1}$

$\bar{f}(s) = \dfrac{s+1}{(s+1)^2+1}$

from. Laplace integral

$= \dfrac{1}{s} \left(\dfrac{s+1}{s^2+2s+2} \right)$

$L\left\{ \int_0^t e^{-t} \cos t \, dt \right\} = \dfrac{s+1}{s^3+2s^2+2s}$

$L\left\{\int_0^t t e^{-t} \sin 4t \, dt\right\}$

$f(t) = e^{-t} \sin 4t$

$\bar{f}(s) = L\{e^{-t} f(t)\} = \dfrac{4}{(s+1)^2+16}$

$L\{t\, f(t)\} = (-1)^n \dfrac{d}{ds}\, \bar{f}(s)$

$= \dfrac{d}{ds} \dfrac{4}{s^2+2s+17}$

$= \dfrac{0 - 4(2s+2)}{(s^2+2s+17)^2}$

$= \int_0^t \dfrac{-8s-8}{(s^2+2s+17)^2}$

$\dfrac{1}{s} \times \dfrac{-8(s+1)}{(s^2+2s+17)^2}$

Lalplace transform of $t^n . f(t)$

- Multiply by "t": If $f(t)$ is sectionally continuing and exponenctual order if

$L\{f(t)\} = \bar{f}(s)$ then

$L\{t\, f(t)\} = \dfrac{-d}{ds} \bar{f}(s)$.

- Multiplication by "t^n": If $L\{f(t)\} = \bar{f}(s)$

then. $L\{t^n f(t)\} = (-1)^n \dfrac{d^n}{ds^n} \bar{f}(s)$ where

$n = 1, 2, 3, \ldots$

1) Evaluate $L\{t \sin 3t \cos 2t\}$

$[2\sin A\cos B = \sin A+B + \sin A-B]$

$$L\{t f(t)\} = \frac{-d}{ds}\bar{f}(s)$$

$$L\{f(t)\} = L\{\sin 3t \cos 2t\}$$

$$\bar{f}(s) = \frac{1}{2} L\{\sin 5t + \sin t\}$$

$$\frac{1}{2}\left[\frac{5}{s^2+2s} + \frac{1}{s^2+1}\right]$$

$$\bar{f}(s) = \frac{1}{2}\left[\frac{5}{s^2+2s} + \frac{1}{s^2+1}\right]$$

$$L\{t \sin 3t \cos 2t\} = \frac{-d}{ds}\frac{1}{2}\left[\frac{5}{s^2+2s} + \frac{1}{s^2+1}\right]$$

$$\frac{vu' - uv'}{v^2}$$

$$= \frac{-1}{2}\frac{d}{ds}\left[\frac{5}{s^2+2s}\right] + \frac{-1}{2}\frac{d}{ds}\left[\frac{1}{s^2+1}\right]$$

cont...

$$= \frac{-1}{2}\frac{(s^2+2s)(0) - s(2s)}{(s^2+2s)^2}$$

$$\frac{-1}{2}\frac{(s^2+1)0 - 1(2s)}{(s^2+1)^2}$$

$$= \frac{-1}{2}\frac{-2s^2}{(s^2+2s)^2} - \frac{1}{2}\frac{(-2s)}{(s^2+1)^2}$$

$$= \frac{s^2}{(s^2+2s)^2} + \frac{2s}{2}\frac{1}{(s^2+1)^2} = L\{t \sin 5t \cos 2t\}$$

$$\frac{-1}{2}\left[\frac{(s^2+25)0-5(2s)}{(s^2+25)^2}+\frac{(s^2+1)0-1(2s)}{(s+1)^2}\right]$$

$$\frac{-1}{2}\left[\frac{-5(2s)}{(s^2+25)^2}+\frac{-2s}{(s^2+1)^2}\right]$$

$$\therefore L\{t \sin 3t \cos 2t\} = \frac{5s}{(s^2+25)^2}+\frac{s}{(s^2+1)^2}$$

$$L\{t^2 \sin 2t\}$$

$$f(t) = \sin 2t$$

$$\bar{f}(s) = \frac{2}{s^2+4}$$

$$L\{t^2 f(t)\} = (-1)^2 \frac{d^2}{ds^2} \bar{f}(s)$$

$$= \frac{d^2}{ds^2}\frac{2}{s^2+4}$$

$$= \frac{d}{ds}\frac{(s^2+4)0-2(2s)}{(s^2+4)^2}$$

$$= \frac{d}{ds}\left[\frac{-4s}{(s^2+4)^2}\right]$$

$$s^4+2s^2+16$$

$$(s^2+4)$$

$$\frac{-4-4s^2}{(s^2+4)^3}$$

$$= \frac{(s^2+4)^2(-4)-(-4s)(4s^3+2s)}{(s^2+4)^4}$$

$$\frac{-4(1-s^2)}{(s^2+4)^3}.$$

$$= \frac{-16s^4-16s^2-4s^4-8s^2-64}{(s^2+4)^4}$$

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Q Find the laplace Transform of $(t^2 e^{2t} \sin t)$

$L(t^2 e^{2t} \sin t) =$

$$L(\sin t) = \frac{1}{s^2+1}$$

$$L(e^{2t} \sin t) = \frac{1}{(s-2)^2+1}$$

$$L(t^2 e^{2t} \sin t) = \frac{(-1)^2 d^2 \cdot f(t)}{ds^2}$$

$$= \frac{d^2}{ds^2}\left[\frac{1}{(s-2)^2+1}\right]$$

$$= \frac{d^2}{ds^2}\left[\frac{1}{s^2-4s+5}\right]$$

$$\frac{(s^2-2s+5)\,0 - 1(2s-2)}{(s^2-4s+5)^2}$$

$$\frac{d}{ds}\frac{-2s+2}{(s^2-4s+5)^2} \qquad \cancel{0-1(-2s+2)}$$

$$= 2\left[\frac{-\ 12s}{\begin{matrix} s^4 - 2s^3 + 5s^2 \\ -2s^3 + 4s^2 - 5s \\ +5s^2 - 10s + 25 \end{matrix}}\right]$$

$$= 2\left[\frac{1 - 5}{s^4 - 4s^3 + 14s^2 - 130}\right]$$

$$2\left[\frac{s^4-4s^3+14s^2-130-\left[(1+s)(4s^3-12s^2+28s\right]}{(s^4-4s^3+14s^2-130)^2}\right]$$

-4
$+4$
-72

$\cancel{5s^4-12s^3+30s^2}$.

$$2\left[\frac{s^4-4s^3+14s^2-130-4s^3+12s^2+28s+8s^4+12s^3+28s^2}{(s^4-4s^3+14s^2-130)^2}\right]$$

$+14$
-12
$+28$

$$\cancel{2\left[\frac{-3s^4+4s^3-3s^2-28s-130}{(s^4-4s^3+14s^2-130)^2}\right]}$$

$$\cancel{2\left[\frac{5s^4-20s^3+54s^2-28s-130}{(s^4-4s^3+14s^2-130)^2}\right]}$$

$$2\left[\frac{9s^4-32s^3+82s^2-28s-130}{(s^2-4s+5)^4}\right]$$

$$=\frac{2(3s^2-12s+11)}{(s^2-4s+5)^3}$$

9 $L(t\,e^{-t}\cdot\sin 2t\cos 2t)$

$$L\int_0^t (t\,e^{-t}\cdot\sin 2t\,dt)$$

* Division by "t" :

If $L\{f(t)\} = \bar{f}(s)$

Then $L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} \bar{f}(s)\,ds$

Provided, the integral exists.

Proof :-

RHS :- $\int_s^{\infty} \bar{f}(s)\,ds = \int_s^{\infty}\int_0^{\infty} e^{-st} f(t)\,dt\,ds$

$$= \int_0^{\infty}\left[\int_s^{\infty} e^{-st} f(t)\,ds\right] dt$$

$$= \int_0^{\infty}\left[f(t) \int_s^{\infty} e^{-st}\,ds\right] dt$$

$$= \int_0^{\infty} f(t)\left[\frac{e^{-\infty t}}{-t} - \frac{e^{-st}}{-t}\right] dt$$

$$= \int_0^{\infty} f(t)\,\frac{e^{st}}{t}\,dt$$

$L\left\{\frac{f(t)}{t}\right\} = \int_0^{\infty} e^{-st}\,\frac{f(t)}{t}\,dt \quad L\left\{\frac{f(t)}{t}\right\}$

= LHS

Q. $L\left\{\frac{\sin t}{t}\right\}$

$$\left[\because L\{\sin t\} = \frac{1}{s^2+1}\right]$$

$f(s) =$

$$L\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{1}{s^2+1}\,ds$$

$$= \left[\tan^{-1} s\right]_s^\infty$$

$$L\left\{\frac{\sin t}{t}\right\} = \frac{\pi}{2} - \tan^{-1}(s).$$

Q) $L\left\{\frac{\sin 3t \cdot \cos t}{t}\right\}$

$$L\left\{\frac{1}{2}\frac{\sin 4t}{2t} + \frac{\sin 2t}{2t}\right\}$$

$$\because L\{\sin 4t\} = \frac{4}{s^2+16} \qquad \frac{2}{s^2+4}$$

$$L\left\{\frac{\sin 4t}{2t}\right\} = \frac{1}{2}\int_s^\infty \frac{4}{s^2+16}$$

$$= \frac{1}{2}\left[\tan^{-1}\left(\frac{s}{4}\right) + \tan^{-1}\left(\frac{s}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{s}{4}\right) + \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{2}\right)\right]$$

$= \frac{\pi}{2} - \tan^{-1}\frac{s}{4} - \tan^{-1}\frac{s}{2}$

Q) evaluation of integrals by Laplace transforms:

Q) using L. Transforms evaluate integrals.

$$\int_0^\infty e^{-3t} \sin t\, dt.$$

$= \quad f(t) = \sin t$

$$L(f(t)) = \frac{1}{1+s^2}$$

$$\int_0^\infty e^{-3t} \sin t = \frac{1}{1+s^2}$$

put $s = 3$

$$= \frac{1}{1+3^2}$$

$= 1$.

② $\int_0^\infty t^2 e^{-4t} \sin 2t\, dt = {}^{11}/_{500}$.

$f(t) = t^2 \sin 2t$

$L(f(t)) = L(t^2 \sin 2t)$

$$= (-1)^2 \frac{d^2}{ds^2}\left(\frac{2}{s^2+4}\right)$$

$$= 2 \frac{d}{ds}\left(\frac{d}{ds}\left(\frac{1}{s^2+4}\right)\right)$$

$$= 2 \frac{d}{ds}\left[-\frac{2s}{(s^2+4)^2}\right]$$

$$= -4\left[\frac{(s^2+4)^2(1) - (2(s^2+4)(2s)(s))}{(s^2+4)^4}\right]$$

$$= -4\,\cancel{(s^2+4)}\left[\frac{(s^2+4) - 4s^2}{(s^2+4)^{\cancel{4}\,3}}\right]$$

$$= -4\left[\frac{-3s^2+4}{(s^2+4)^3}\right]$$

Now,

$$\int_0^\infty t^2 e^{-4t} \sin 2t\,dt = \qquad s=4$$

$$\frac{-4(-3(4)^2+4)}{(4^2+4)^3} = \frac{4}{500}$$

3) using L.T $\int_0^\infty t e^{-3t} \sin t\,dt = 3/50$.

$$\frac{1}{1+(s+3)^2}$$

$$-\frac{d}{dt}\frac{1}{(s+3)^2+1}$$

* unit step function:-

→ This function denoted by $u(t-a)$ or $H(t-a)$ is defined as follows

i.e; $u(t-a)$ or $H(t-a) = \begin{cases} 0 & t < a \\ 1 & t > a \end{cases}$

i.e $L(u(t-a)) = \frac{e^{-as}}{s}$ //.

* unit impulse function (or) direct delta function

→ unit impulse function is discontinous function on the function $f(t-a)$ is defined as $f(t-a) =$

$L f(t-a) = e^{-as}$ //.

* Laplace transform of periodic function:

A function $f(t)$ is said to be PF. If and only if $f(t+T) = f(t)$ where, T is periodic of $f(t)$. The smallest +ve value of T for which this equation is true for every value of t. is called periodic function.

Then,

$$L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt.$$

√problems:-

1. $f(t) = \begin{cases} k & 0<t<a \\ -k & a<t<2a \end{cases}$

$$L(f(t)) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt.$$

$$= \frac{1}{1-e^{-s(2a)}} \int_0^{2a} e^{-st} f(t)\,dt.$$

$$= \frac{1}{1-e^{-2sa}} \left[\int_0^a e^{-st} f(t)\,dt + \int_a^{2a} e^{-st} f(t)\,dt\right]$$

$$= \frac{1}{1-2^{-2sa}} \left[\int_0^a e^{-st}(k)\,dt + \int_0^{2a} e^{-st}(-k)\,dt\right]$$

$$= \frac{1}{1-e^{-2sa}} \left[k\int_0^a e^{-st}\,dt - k\int_a^{2a} e^{-st}\,dt\right]$$

$$= \frac{1}{1-e^{-2as}} \left[k\left[\frac{e^{-st}}{-s}\right]_0^a - k\left[\frac{e^{-st}}{-s}\right]_a^{2a}\right]$$

$$= \frac{1}{1-e^{-2as}} \left[k\left[\frac{e^{-s(a)}}{-s} - \frac{1}{-s}\right] - k\left[\frac{e^{-2as}}{-s} - \frac{e^{-sa}}{-s}\right].\right.$$

$$\Rightarrow \frac{K}{s(1-e^{-2as})}\left[-e^{-s(a)}+1+e^{-2as}-e^{-as}\right]$$

$$\frac{K}{s(1-e^{-2as})}\left[e^{-2as}+1-2e^{-as}\right]$$

$$\frac{K}{s(1-e^{-2as})}\left[(1-e^{-as})^2\right]$$

$$=\frac{K(1-e^{-as})}{s}$$

② Find $L[|\sin t|]$

$$f(t)=|\sin t|$$
$$=\sin t \quad 0<t<\pi.$$

$$L(f(t))=\frac{1}{1-e^{-sT}}\int_0^T e^{-sT} f(t)\,dt.$$

$$=\frac{1}{1-e^{-s\pi}}\int_0^\pi e^{-st} f(t)\,dt.$$

$$=\frac{1}{1-e^{-\pi s}}\int_0^\pi e^{-st}\sin t\,dt.$$

$$=\frac{1}{1-e^{-\pi s}}\left[\frac{e^{-sT}}{1+s^2}(-s\sin t-\cos t)\right]_0^\pi$$

③ $= \frac{1}{1-e^{-\pi s}}\left[\frac{e^{-st}}{1+s^2}(-s\sin t - \cos t)\right]_0^{\pi}$

If $\bar{f}(s)$ is the laplace transform of the function $f(t)$ then $f(t)$ is called Inverse laplase of $\bar{f}(s)$ & it is denoted by $L^{-1}(\bar{f}(s)) = f(t)$.

Table of Inverse laplace transforms:-

S No.	$\bar{f}(s)$		$L^{-1}\{\bar{f}(s)\} = f(t)$
1	$\frac{1}{s}$		1.
2	$\frac{1}{s^{n+1}}$	$n = +ve$	$\frac{t^n}{n!}$
3	$\frac{1}{s^{n+1}}$	$n > 1$	$\frac{t^n}{\Gamma(n+1)}$
4	$\frac{1}{s-a}$		e^{at}

5	$\frac{1}{s+a}$	e^{-at}
6	$\frac{1}{s^2+a^2}$	$\frac{1}{a}\sin at$
7.	$\frac{s}{s^2+a^2}$	$\cos at$
8.	$\frac{1}{s^2-a^2}$	$\frac{1}{a}\cosh at$
9.	$\frac{s}{s^2-a^2}$	$\cosh at$
10.	$\frac{1}{(s-a)^2+b^2}$	$\frac{1}{b}e^{at}\sin bt$
11.	$\frac{s-a}{(s-a)^2+b^2}$	$e^{at}\cos bt$
12.	$\frac{1}{(s-a)^2-b^2}$	$\frac{1}{b}e^{at}\sinh bt$
13.	$\frac{s-a}{(s-a)^2-b^2}$	$e^{at}\cosh at$
14.	$\frac{2as}{\cancel{\sin}(s^2+a^2)^2}$	$t\sin at$
15.	$\frac{s^2-a^2}{(s^2+a^2)}$	$t\cos at$

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1) find the inverse laplace of $1/s^3$

$L^{-1}(1/s^3)$.

$\therefore \left\{ L^{-1} \frac{1}{s^{n+1}} = \frac{t^n}{n!} \right\}$.

$\therefore L^{-1} \frac{1}{s^{n+1}} = \frac{t^n}{\Gamma(n+1)}$

$n+1 = 3$

$n = 3-1$.

$\boxed{n=2}$.

$L^{-1} \frac{1}{s^2+1} = \frac{t^2}{2!}$

$L^{-1}\left(\frac{1}{s^3}\right) = t^2/2$

) $L^{-1}\left(\frac{3(s^2-2)^2}{2s^5}\right)$

$\frac{3}{2} L^{-1}\left[\frac{(s^2-2)^2}{s^5}\right]$

$\frac{3}{2} L^{-1}\left[\frac{s^4+4-4s^2}{s5}\right]$

$\frac{3}{2} L^{-1}\left(\frac{\cancel{s^4}}{s^{\cancel{5}1}} + \frac{4}{s^5} - \frac{4s^{\cancel{2}}}{s^{\cancel{5}3}}\right)$

$\frac{3}{2}\left[L^{-1}\left(\frac{1}{s^1}\right) + 4L^{-1}\frac{1}{s5} - 4L^{-1}\left(\frac{1}{s^3}\right)\right]$

$\left(\frac{t^2}{2}\right)$]

$\frac{3}{2}\left(1 + \cancel{4}\frac{t^4}{\cancel{4}\times 6} - \cancel{4}^{2}\left(\frac{t^2}{\cancel{2}}\right)\right)$

$= 3/2\left(1 + t^4/6 - 2t^2\right)$

3) $\mathcal{L}^{-1}\left[\frac{4}{(s+1)(s+2)}\right]$.

let

$$\frac{4}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$4 = A(s+2) + B(s+1)$$

$$= As + 2A + Bs + B$$

$$0s + 4 = s(A+B) + (2A+B).$$

$$A+B=0 \quad 2A+B=4$$

$$A=4$$
$$B=-4.$$

$$\rightarrow 4 = A(-1+2) + B(-1+1)$$

$$\boxed{A=4}$$

$$4 = A(-2+2) + B(-2+1)$$

$$= A(0) + B(-1)$$

$$\boxed{B=-4}$$

$$\frac{4}{(s+1)(s+2)} = \frac{4}{s+1} - \frac{4}{s+2}$$

Now,

$$\mathcal{L}^{-1}\left[\frac{4}{(s+1)(s+2)}\right] = \mathcal{L}^{-1}\left[\frac{4}{s+1} - \frac{4}{s+2}\right]$$

$$= \mathcal{L}^{-1}\left(\frac{4}{s+1}\right) - \mathcal{L}^{-1}\left(\frac{4}{s+2}\right)$$

$$= 4\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) - 4\mathcal{L}^{-1}\left(\frac{1}{s+2}\right)$$

$= 4e^{-1(t)} - 4e^{-2t}$

$= 4e^{-t} - 4e^{-2t}$

8) $L^{-1}\left[\frac{5s-2}{s^2(s+2)(s-1)}\right]$

$$\frac{5s-2}{s^2(s+2)(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s+2)} + \frac{D}{s-1}$$

$$5s-2 = As(s+2)(s-1) + B(s+2)(s-1) + C(s^2(s-1)) + D(s^2(s+2))$$

when $s=0 \Rightarrow 5(0)-2 = A(0) + B(0+2)(0-1) + C(0) + D(0)$.

$B(-2) = -2$

$\boxed{B=1}$

$s=-2 \Rightarrow 5(-2)-2 = A(0) + B(0) + C(-2)^2(-2-1) + D(0)$.

$-12 = -12C$

$\boxed{C=1}$

$s=1 \rightarrow 5(1)-2 = A(0) + B(0) + C(0) + D(3)$.

$3 = 3D$

$\boxed{D=1}$

$A + C + D = 0$

$A + 1 + 1 = 0$

$\boxed{A=-2}$

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Now

$$L^{-1}\left(\frac{5s-2}{s^2(s+2)(s+1)}\right) = L^{-1}\left(\frac{-2}{s}+\frac{1}{s^2}+\frac{1}{s+2}+\frac{1}{s-1}\right)$$

$$= L^{-1}\left(-2/s\right) + L^{-1}\left(1/s^2\right) + L^{-1}\left(\frac{1}{s+2}\right) + L^{-1}\left(\frac{1}{s-1}\right)$$

$$= -2 + t + e^{-2t} + e^{-t}$$

Old
Arbitary
King

$$L^{-1}\left\{\frac{s^2+2s-4}{(s^2+9)\ (s-5)}\right\}$$

A) $L^{-1}\left\{\frac{1}{34}\left[\left(\frac{3s}{s^2+9}\right)+\left(\frac{83}{s^2+9}\right)+\frac{31}{s-5}\right]\right\}$

$$\frac{1}{34}\left[3\left(L^{-1}\left\{\frac{s}{s^2+9}\right\}\right)+83\left(L^{-1}\left\{\frac{1}{s^2+9}\right\}\right)+31\left(L^{-1}\left\{\frac{1}{s-5}\right\}\right)\right]$$

$$= \frac{1}{34}\left[3\cos 3t + 83\ \frac{\sin 3t}{3} + 31\left(e^{5t}\right)\right]$$

$$= \frac{3\cos 3t}{34} + \frac{83\sin 3t}{102} + \frac{31e^{5t}}{34}$$

$$\therefore L^{-1}\left\{\frac{s^2+2s-4}{(s^2+9)\ (s-5)}\right\} = \frac{3\cos 3t}{34} + \frac{83\sin 3t}{102} + \frac{31e^{5t}}{34}.$$

9) $L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+25)}\right\}$

$L^{-1}\left\{\frac{1}{21}\left[\frac{1}{s^2+4} - \frac{1}{s^2+25}\right]\right\}$

$\frac{1}{21}\left(L^{-1}\left\{\frac{1}{s^2+2^2}\right\} - L^{-1}\left\{\frac{1}{s^2+5^2}\right\}\right)$

$\frac{1}{21}\left(\frac{1}{2}\sin 2t - \frac{1}{5}\sin 5t\right)$

$\therefore L^{-1}\left\{\frac{S^2}{(S^2+4)(S^2+25)}\right\} = \frac{\sin 2t}{42} - \frac{\sin 5t}{105}$

10/07/2023

* **1st Shifting Theorem :-**

If $L^{-1}(\bar{f}(s)) = f(t)$ then

$L^{-1}\{\bar{f}(s-a)\} = e^{at} f(t).$

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8)

$$L^{-1}\left\{\frac{1}{s^2+2s+5}\right\}$$

$$L^{-1}\left\{\frac{1}{(s+1)^2+4}\right\}$$

$$L^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\} = \frac{1}{2} e^{-t} \sin 2t$$

from formula.

(or)

$$L^{-1}\left\{\frac{1}{(s+1)^2+4}\right\} =$$

$s-a$

$a = -1$

from 1st Shifting,

$$e^{-t} L^{-1}\left\{\frac{1}{s^2+2^2}\right\}$$

$$= e^{-t}\frac{1}{2}\sin 2t$$

9)

$$L^{-1}\left(\frac{3s-2}{s^2-4s+20}\right)$$

$3s-6+4$

$3(s-2)+4$

$$L^{-1}\left\{\frac{3s-2}{(s-2)^2+4^2}\right\} = L^{-1}\left\{\frac{3(s-2)+4}{(s-2)^2+4^2}\right\}$$

$s-a$

$a = 2$

First Shifting :-

$$e^{2t} L^{-1}\left\{\frac{3s+4}{s^2+4^2}\right\}$$

$$e^{2t}\left[3\left(L^{-1}\left\{\frac{s}{s^2+4^2}\right\}\right)+4\left(L^{-1}\left\{\frac{1}{s^2+4^2}\right\}\right)\right]$$

$$= e^{2t}\left(3\cos 4t + 4\left(\frac{1}{4}\sin 4t\right)\right)$$

$$= e^{2t}(3\cos 4t + \sin 4t)$$

9) $L^{-1}\left[\frac{s+2}{s^2-6s+8}\right]$

$$L^{-1}\left\{\frac{s+2}{(s-3)^2-1}\right\}$$

$$L^{-1}\left\{\frac{(s-3)+5}{(s-3)^2-1}\right\}$$

$s-a$

$a=3$

First Shifting :-

$$e^{3t} L^{-1}\left\{\frac{s+5}{s^2-1^2}\right\}$$

$$= e^{3t}\cosh t + 5e^{3t}\sinh t$$

HW

9) $L^{-1}\left\{\frac{s+3}{s^2-10s+29}\right\}$

$(s-5)^2+4$

9) $L^{-1}\left\{\frac{s^2}{s^4+4a^4}\right\}$

$\frac{s^2}{(s^2+2a^2)^2-4a^2s^2}$

① $L^{-1}\left\{\frac{s-5+8}{(s-5)^2+4}\right\}$

$(s-a)$

$a=5$

First Shifting :-

$e^{5t}\, L^{-1}\left\{\frac{s+8}{s^2+2^2}\right\}$

$e^{5t}(\cos 2t)$

$+\frac{8.e^{5t}\sin 2t}{2}$

$\boxed{= e^{5t}(\cos 2t + 4\sin 2t)}$

② $\frac{s^2+2a^2-2a^2}{(s^2+2a^2)^2 - 4a^2s^2 - 8a^4 + 8a^4}$

$(s-a)$,

$a = -2a^2$

$= \boxed{\frac{(s^2+2a^2)-2a^2}{(s^2+2a^2)^2 - 4a^2(s^2+2a^2) + 8a^4}}$

First Shifting :-

$e^{-2a^2t}\, L^{-1}\left\{\frac{s^2-2a^2}{s^4 - 4a^2s^2 + 8a^4}\right\}$

$e^{-2a^2t}\, L^{-1}\left\{\frac{s^2-2a^2}{(s^2)^2 - 2(s^2)(2a^2) + 4a^4 + 4a^2}\right\}$

$e^{-2a^2t}\, L^{-1}\left\{\frac{s^2-2a^2}{(s^2-2a^2)^2 + 4a^2}\right\}$

First Shifting :-

$e^{-2a^2t}\, e^{2a^2t}\, L^{-1}\left\{\frac{s^2}{s^4+4a^2}\right\}$

$= \frac{\sin^2 2at}{2a}$

07/12 Convolution Theorem :

· Let $f(t)$ and $g(t)$ be two functions defined as

$$f(t) * g(t) = \int_0^{t} f(t) \cdot g(t-u)\, dt$$

$$= \int_0^{t} f(u) \cdot g(\cancel{u-t}\; t-u)\, du.$$

* or let $L^{-1}\{\bar{f}(s)\} = f(t)$

and

$L^{-1}\{\bar{g}(s)\} = g(t)$, Then.

$$L^{-1}\{\bar{f}(s) \cdot \bar{g}(s)\} = f(t) * g(t)$$

$$= \int_0^{t} f(u) \cdot g(t-u) \cdot du.$$

Q) Using Convolution Theorem find

$$L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$$

$$L^{-1}\left\{\frac{1}{s+a} \times \frac{1}{s+b}\right\}.$$

Let $f(t) = L^{-1}\left(\frac{1}{s+a}\right)$

$= e^{-at} = f(t)$

$g(t) = L^{-1}\left(\frac{1}{s+b}\right)$

$= e^{-bt}$

$= g(t)$

By convolution Theorem

$$f(t) \cdot g(t) = \int_0^{u} f(t)\, g(t-u)\, dt$$

$$= \int_0^{u} e^{-at} e^{-b(t-u)}\, dt$$

$$= \int_0^{u} e^{-t(a+b)+bu}\, dt$$

$$= \int_0^{u} e^{-t(a+b)}\, e^{bu}\, dt$$

$$= e^{bu} \int_0^{u} e^{-t(a+b)}\, dt$$

$$= e^{bu} \left[\frac{e^{-t(a+b)}}{-(a+b)} \right]_0^{u}$$

$$= e^{bu} \left(\frac{e^{-u(a+b)}}{-(a+b)} \right) - e^{bu} \frac{e^{-0}}{-(a+b)}$$

$$= e^{bu} \left(\frac{1}{a+b} - \frac{e^{-u(a+b)}}{a+b} \right)$$

9) $L^{-1} \left\{ \frac{1}{s(s^2+4)} \right\}$

$f(t) = L^{-1}\left\{\frac{1}{s}\right\} = 1$

$g(t) = L^{-1} \left\{ \frac{1}{s^2+2^2} \right\} = \frac{\sin 2t}{2}$

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$$f(t) \cdot g(t) = \int_0^u 1 \cdot \frac{\sin 2(t-u)}{2} dt .$$

$2(t-u) = v$

$dt = \frac{dv}{2}$

$$= \frac{1}{2} \int_0^u \sin 2(t-u)\, dt$$

$$\frac{1}{2} \left[-\frac{\cos 2(t-u)}{2} \right]_0^u$$

$$= \frac{1}{2} \left[\frac{\cos 2u}{2} - 1 \right]$$

$$= \frac{\cos 2u - 1}{4}$$

$$\frac{1}{2} \int_0^t \sin 2(t-u)\, du$$

$$\frac{1}{2} \cdot \left[-\frac{\cos 2(t-u)}{2} \right]_0^t$$

$$\frac{1}{2} \left[+\frac{1}{2} - \frac{\cos 2t}{2} \right]$$

$$\frac{1 - \cos 2t}{4}$$

9) Using Convalation Theorem Evaluate

$$L^{-1} \left\{ \frac{1}{s.(s^2+2s+2)} \right\}$$

$g(t)$ $f(t) = L^{-1}\left(\frac{1}{s}\right) = 1$

$f(t)$ $g(t) = L^{-1}\left(\frac{1}{s^2+2s+2}\right)$

$$\frac{1}{(s+1)^2+1}$$

$s - a$

$a = -1$

$$= e^{-t} \sin t$$

Convolation Theorem $\int_0^u$ $\cancel{1}$. $e^{-t} \sin t\, dt$

$\boxed{\int e^{ax} \sin bx = \frac{e^{ax}}{a^2+b^2} [a \sin x - b \cos x]}$

$$\left[-e^{-t} \frac{(\sin t + \cos t)}{2} \right]_0^u$$

$$= \frac{1}{2} - \frac{e^{-u}(\sin u + \cos u)}{2}$$

$$\therefore L^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\} = \frac{1}{2} - \frac{e^{-u}(\sin u + \cos u)}{2}$$

Q) H.W find $L^{-1}\left\{\frac{1}{(s-2)(s+2)^2}\right\}$

$$g(t) = L^{-1}\left\{\frac{1}{s-2}\right\} = e^{2t}$$

$$f(t) = L^{-1}\left(\frac{1}{(s+2)^2}\right) = \frac{e^{-2t}\, t^2}{2}$$

$$f(t) * g(t) = \int_0^u \frac{e^{-2t}\, t^2}{2} \cdot e^{2(t-u)}\, dt$$

$$= \frac{e^{-u}}{2} \int_0^u t^2\, dt$$

$$\frac{e^{-u}}{2}\left[\frac{t^3}{3}\right]_0^u$$

$$= \frac{e^{-u} u^3}{6}$$

8) $L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right]$

$f(t) = L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{\sin at}{a}$

$g(t) = \frac{\sin at}{a}$

$f(t) \cdot g(t) = \int_0^u f(t) \cdot g(u-t)\, dt$

$\int_0^u \frac{\sin at}{a} \frac{\sin \overset{a(u-t)}{\cancel{(u-t)t}}}{a}\, dt$

$\frac{1}{a^2} \int_0^u \sin at \ \sin \overset{a(u-t)}{\cancel{(u-t)t}}\, dt$

$= \frac{1}{a^2}\left[\frac{\sin a(2t-u) - 2a \cos u\, at}{4a}\right]_0^u$

$\frac{1}{2a^3}$ $\sin at - at \cos at$

$= \frac{\sin au - au \cos au}{2a^3}$

12/01/23

$$\frac{1}{(s^2+4)(s+1)^2}$$

$f(t)$ $L^{-1}\left(\frac{1}{s^2+4}\right) = \frac{\sin 2t}{2}$

$g(t)$ $L^{-1}\left\{\frac{1}{(s+1)^2}\right\} = \frac{e^{-t} t^2}{2}$

$s-a$ $\left(\frac{1}{s^2}\right)$

$a = -1$

$$f(t) \cdot g(t) = \int_0^u f(t)\, g(t-u)\, dt$$

$$\int_0^u \frac{e^{-t} t^2}{2} \cdot \frac{\sin 2(t-u)}{2} dt$$

$\frac{2t}{2}$

$$\frac{1}{4}\left[\int_0^u e^{-t} t^2 \sin 2(t-u)\, dt\right]$$

e^{-1}

$\frac{1}{4}$

$$L^{-1}\left(\frac{1}{(s^2+4)(s+1)} * \frac{1}{(s+1)}\right)$$

STORIES MAKE PEOPLE IMMORTAL

$$\int_0^u \frac{\sin 2t}{2} e^{-a(t-u)} dt$$

$$\frac{e^{au}}{2} \int_0^u \sin 2t \, e^{-at} \, dt$$

$$= \frac{e^{au}}{2}\left[\frac{-e^{au}(a\sin 2u + 2\cos 2u)}{a^2+4} + \frac{2}{a^2+4}\right]$$

$$\frac{1}{s(s-1)} \quad \left(\frac{1}{s+2}\right)$$

$$\int_0^t L^{-1}\left(\frac{1}{s}\right)\left[L^{-1}\left(\frac{1}{s-1}\right)\right] du$$

$$\int e^{at} \quad 1 \quad dt$$

$$\left[\frac{e^{at}}{a}\right]_0^t$$

$$\frac{e^{at}}{a} - \frac{1}{a} \qquad \frac{1}{s+1}$$

$$\frac{e^{at}-1}{a} \qquad \frac{1}{s+1} \qquad au-at-t$$

$$\frac{e^{-t}\cdot t}{a} - \frac{t}{a}$$

$$\frac{e^{at}-1}{a} \qquad e^{-a(t-t)} \qquad \frac{e^{-t}u}{a} - \frac{u}{a}$$

$$\boxed{= \frac{t}{a}\left(e^{-t}-1\right)}$$

13/07/23

Inverse Laplace Transform of Derevative

If $L^{-1}\{\overline{f}(s)\} = f(t)$ then

$L^{-1}\,\overline{f^{(n)}}(s) = (-1)^n t^n f(t)$.

where $\overline{f^{(n)}}(s) = \frac{d^n}{ds^n} f(s)$

$\boxed{\text{note : } f(t) = -\frac{L^{-1}\,\overline{f'}(s)}{t}}$

Q) find $L^{-1}\left\{\log\left(1+\frac{1}{s^2}\right)\right\}$

$\overline{f}(s) = \log\left(\frac{s^2+1}{s^2}\right)$

$\overline{f}(s) = \log(s^2+1) - 2\log s$

$\overline{f'}(s) = \frac{2s}{s^2+1} - \frac{2}{s}$

$L^{-1}\,\overline{f'}(s) = -\frac{L^{-1}\left(\overline{f'}(s)\right)}{t}$

$= \frac{1}{t}\left[L^{-1}\left\{\frac{2s}{s^2+1} - \frac{2}{s}\right\}\right]$

$L^{-1}\left\{\log\left(1+\frac{1}{s^2}\right)\right\} = -\left[\frac{2\cos t - 2}{t}\right]$

find $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$

$\bar{f}(s) = \log(s+1) - \log(s-1)$

$\bar{f}'(s) = \frac{1}{s+1} - \frac{1}{s-1} = \frac{s-1-s-1}{s^2-1}$

$= \frac{-2}{s^2-1}$

$L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\} = \frac{-L^{-1}}{t}\left\{\frac{-2}{s^2-1}\right\}$

$= +\frac{2}{t}\ \cancel{\sinh t}\ -\cosh t$

$= +\frac{2}{t}\left(\frac{e^{t}+e^{-t}}{2}\right)$

$= +\frac{e^{t}+e^{-t}}{t}.$

find $L^{-1}\left(\cot^{-1}(s)\right)$

$f(s) = \frac{-L^{-1}\bar{f}'(s)}{t}$

$\bar{f}(s) = \cot^{-1}(s)$

$= \frac{-1}{1+s^2}.$

$= L^{-1}\left\{\frac{-1}{1-s^2}\right\}$

$f'(s) = -\cancel{\sin t}\ \frac{-1}{1-s^2}$

$L^{-1}\{f'(s)\} = -\sin t$

$$f(t) = -L^{-1} \frac{f'(s)}{t}$$

$$= \frac{\sin t}{t}.$$

Q) $L^{-1}\left\{ \log(s^2+4) - \log(s^2+9) \right\}$

$$f'(s) = \frac{2s}{s^2+4} - \frac{2s}{s^2+9}$$

$$L^{-1} f'(s) = \frac{-2\cos 2t + 2\cos 3t}{t}$$

$$L^{-1}\left\{ \log\left(\frac{s^2+16}{\cancel{s^2}}\right) - \log(s^2) \right.$$

$$f(s) = \frac{2s}{s^2+16} - \frac{2s}{s^2}$$

$$L^{-1} f'(s) = \frac{-(2\cos 4t - 2)}{t}$$

$$\frac{2}{t} - \frac{2\cos 4t}{t}$$

$$L^{-1} \log\left(1+\frac{16}{s^2}\right) = \frac{2-2\cos 4t}{t}$$

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